

Momentum of a body. The momentum of a body is equal to the mass of the body times its velocity i.e.

$$\vec{p} = m\vec{v}$$

where m is the mass of the body and v is its velocity.

Momentum is a vector quantity, making its direction a necessary part of the data. To define the momentum in our three-dimensional space completely, one needs to specify its three components in x, y and z direction. The momentum of a system of more than one particle is the vector sum of the individual momenta:

$$\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \dots = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \dots$$

Impulse of a force. The impulse of a force is equal to the force times the length of time the force acts i.e.

The 2nd Newton's law of mechanics can be written in a form which states that the rate of the change of the system's momentum with time is equal to the sum of the external forces acting on this system:

$$2) \quad \frac{d\vec{p}}{dt} = \sum \vec{F}$$

where F is the force and t is the length of time the force acts. Impulse is a vector quantity whose direction is that of the force.

From here we can immediately see that when the system is closed (which means that the net external force acting on the system is zero), the total momentum of the system is conserved (constant).

If the force F varies with time (see Fig. 1) the impulse of the force over the time interval $[t_1, t_2]$ is given by

$$3) \quad \text{Impulse} = \int_{t=t_1}^{t=t_2} F(t)dt$$

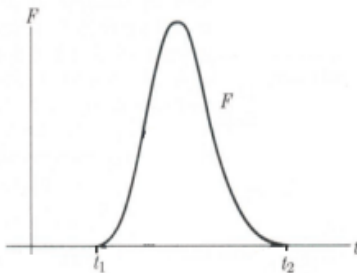


Fig. 1

Relationship between impulse and momentum.

The change of momentum produced by an impulse is numerically equal to the impulse i.e.

$$4) \int_{t=t_1}^{t=t_2} F(t)dt = mv_{t_2} - mv_{t_1}$$

where v_{t_1} and v_{t_2} are the velocities at times $t = t_1$ and $t = t_2$ respectively.

Law of conservation of momentum. In any collision between two or more bodies, the vector sum of the momenta of the bodies after impact is equal to that before impact.

Thus if

m_A, m_B = the masses of bodies A and B

$\mathbf{V}_{A1}, \mathbf{V}_{B1}$ = the velocities of bodies A and B before impact

$\mathbf{V}_{A2}, \mathbf{V}_{B2}$ = the velocities of bodies A and B after impact

then

the momentum of A plus the momentum of B before the collision
= the momentum of A plus the momentum of B after the collision

Or
$$5) \quad m_A \mathbf{V}_{A1} + m_B \mathbf{V}_{B1} = m_A \mathbf{V}_{A2} + m_B \mathbf{V}_{B2}$$

During a collision of two bodies A and B, body A exerts a force F_A on body B and, by Newton's Third Law, body B exerts an equal and opposite reactionary force F_B on body A. Fig. 2 shows these two equal and opposite forces over the interval of impact $[t_1, t_2]$. As a consequence, the impulses of these two forces are equal in magnitude but opposite in sign i.e.

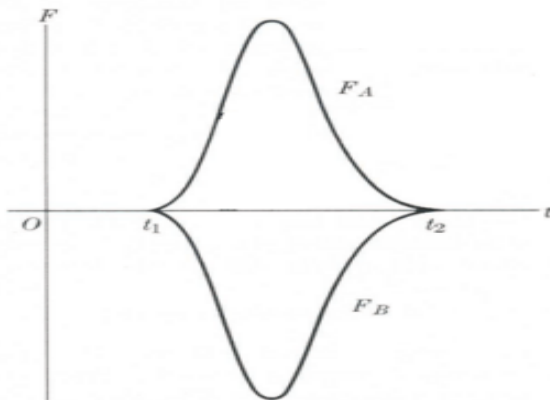


Fig. 2

Impulse of $F_B = -$ Impulse of F_A

Thus

$$m_B \mathbf{V}_{B2} - m_B \mathbf{V}_{B1} = -(m_A \mathbf{V}_{A2} - m_A \mathbf{V}_{A1})$$

Consequently,

$$m_A \mathbf{V}_{A1} + m_B \mathbf{V}_{B1} = m_A \mathbf{V}_{A2} + m_B \mathbf{V}_{B2}$$

In general, in a system of colliding bodies, the sum of the components of momentum along any direction remains unaltered during collision. Consequently, the total momentum of an isolated system with no external forces acting on it is constant in magnitude and direction.

Energy: Another important quantity describing the evolution of the system is its energy. The total energy of a given system is generally the sum of several different forms of energy. Kinetic energy is the form associated with motion, and for a single particle:

$$KE = \frac{mV^2}{2}$$

Here v without the vector symbol stands for the absolute value of the velocity,

$$V = \sqrt{V_x^2 + V_z^2 + V_y^2}$$

In contrast to momentum, kinetic energy is NOT a vector; for a system of more than one particle the total kinetic energy is the algebraic sum of the individual kinetic energies of each particle:

$$KE = KE_1 + KE_2 + KE_3 + \dots +$$

Another fundamental law of physics is that the total energy of a system is always conserved. However within a given system, one form of energy may be converted to another (such as potential energy converted to kinetic in the Pendulum experiment). Therefore, kinetic energy alone is often not conserved.

Elastic and inelastic collisions.

There are two basic kinds of collisions, elastic and inelastic:

In an **elastic** collision, two or more bodies come together, collide, and then move apart again with **no loss in total kinetic energy**. An example would be two identical "superballs", colliding and then rebounding off each other with the same speeds they had before the collision. Given the above example conservation of kinetic energy then implies:

$$\frac{1}{2}m_1 V_{1i}^2 + \frac{1}{2}m_2 V_{2i}^2 = \frac{1}{2}m_1 V_{1f}^2 + \frac{1}{2}m_2 V_{2f}^2; \text{ or } KE_i = KE_f$$

In an **inelastic** collision, the bodies collide and (possibly) come apart again, but now **some**

kinetic energy is lost (converted to some other form of energy). An example would be the collision between a baseball and a bat. If the bodies collide and stick together, the collision is called *completely inelastic*. In this case, all of the kinetic energy *relative to the center of mass of the whole system* is lost in the collision (converted to other forms). Since the collision is inelastic, the initial *KE* is not equal to the final *KE*.

Coefficient of Restitution. For any direct, central impact between two bodies (e.g. spheres) A and B , the coefficient of restitution *e* is defined as

$$6) \quad e = -\frac{V_{A2} - V_{B2}}{V_{A1} - V_{B1}}$$

where

V_{A1} , V_{B1} = the velocities of bodies A and B before impact

V_{A2} , V_{B2} = the velocities of bodies A and B after impact

For a perfectly elastic collision, $e = 1$. For an inelastic collision, $0 < e < 1$. It is assumed that the velocities are all co-linear i.e. that the bodies are both initially and finally moving in the same or opposite directions.

Derivation. Assume that two spheres A and B traveling either directly toward each other or directly away from each other collide in a direct, central impact that is perfectly elastic. Then the total kinetic energy of the two spheres before impact is equal to the total kinetic energy after impact i.e.

$$7) \quad \frac{1}{2} m_A V_{A1}^2 + \frac{1}{2} m_B V_{B1}^2 = \frac{1}{2} m_A V_{A2}^2 + \frac{1}{2} m_B V_{B2}^2$$

Furthermore, the total momentum of the two spheres before impact is equal to the total momentum after impact i.e.

$$8) \quad m_A V_{A1} + m_B V_{B1} = m_A V_{A2} + m_B V_{B2}$$

where we note that 8) is a scalar equation (not a vector equation), the *V*'s representing scalar quantities.

Rearranging equations 7) and 8) we get

$$9) \quad \frac{1}{2} m_A (V_{A1}^2 - V_{A2}^2) = \frac{1}{2} m_B (V_{B1}^2 - V_{B2}^2)$$

$$10) \quad m_A (V_{A1} - V_{A2}) = m_B (V_{B1} - V_{B2})$$

If we now divide 9) by 10) we obtain

$$V_{A1} + V_{A2} = V_{B1} + V_{B2}$$

or, rearranging,

$$11) \quad V_{A1} - V_{B1} = -(V_{A2} - V_{B2})$$

From this equation we can see that the coefficient of restitution

$$6) \quad e = -\frac{V_{A2} - V_{B2}}{V_{A1} - V_{B1}}$$

is equal to one for a perfectly elastic collision, providing the motivation for the definition. For an inelastic collision it is a number between 0 and 1.